Modelling with Context-Dependent Causality

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Abstract

Decision support systems may require the use of existing complex mathematical models. It is desirable to reduce the equations of such a model to an explanatory causal form to support decision making. It is argued here that a combination of extracting information from the user and heuristics to reduce the equations to a suitable form are sufficient to produce a reasonable causal explanation from equations. The method identifies causality in a physical system with or without feedback. The method also allows consistency checking between equations and user generated causal reasoning. This method has been implemented in Hypercard on a Macintosh to provide an accessible graphically oriented decision support tool.

1. Introduction

Experts are often requested to make judgements and give advice in the context of adoption or moderation of future change. For example, political decisions are needed on the human-induced greenhouse warming of the Earth, and an expert adviser is required because of the complexity of the problems. Large mathematical models can be used to simulate the greenhouse effect, but these cannot directly support decision making for reasons to be specified. The need to produce a meaningful causal explanation for justifying an hypothesis is often stated as a fundamental part of an intelligent decision support system [Patil 81].

Our intention is to use existing knowledge to assist the system in explaining and justifying its advice for decision making. The development of methodologies for climate impact advice requires large amounts of knowledge from various resources. This paper focus on the acquisition of knowledge from equations in a mathematical model. Even a simple mathematical model can have very complicated dynamic behaviour [May 76]. However, one of the limits of the mathematical model is that they provide no explicit knowledge on how to perform analysis or to interpret results. When domain experts are asked to explain a result from a numerical simulation, they often describe behaviour as a set of numerical values along some successive time points, but are often unable to give a causal explanation of why things behave as they do [Barr 89].

Statisticians point out that there is no causation in equations [DeLeeuw 85], but if we identify that Y is a “causally dependent” variable, this knowledge helps us to make a link from X to Y rather than from Y to X in the equation $Y = a + bX$. We propose that we use available input and output information from users, if possible, and if this is insufficient we apply the heuristic noted below to assist in identifying reasonable variable dependencies. We then use the dependency characteristic of variables to reconstruct equations to be asymmetric causal equations. An asymmetric equation is one where the variables on the right hand side (RHS) and left hand side (LHS) cannot be exchanged. In this form the LHS of an equation is the dependent variable. Causality is then explicitly represented in the asymmetric causal equations.

Causality is important in understanding, designing and explanation. Iwasaki and Simon, in the development of *causal ordering* [Iwasaki 86, 86a], have worked on the area of deducing causality from equations. In order for causal ordering to produce a “correct” causal structure, each equation must be a *structural equation*, which represents a conceptually distinct *mechanism* in the system. The term "mechanism" describes physical processes as a kind of law and each equation is assigned to one mechanism. Iwasaki stated that unfortunately there is no simple way to identify that an equation is structural [Iwasaki 86, 86a]. Causal ordering *assumes* that equations used in the model are structural equations, and does not provide a method for transforming equations to structural equations. Moreover, a severe shortcoming of causal ordering is in handling feedback [Iwasaki 86, 86a, de Kleer 86, Top 91].

We propose to use input, output and other causal information, as well as the heuristics below to produce "normal" structural equations and "reasonable" causality. We recognise that input and output variables can support the identification of dependencies among variables. Input and output information is available in many domains. The available input and output information can classify variables to be independent or dependent variables within equations. However if the user knows some of the causality in the system, which will often be the case, the method also uses this causality to help determine reasonable causality for the remainder of the system. As well, the heuristics we propose appear to overcome the practical deficiencies of causal ordering in handling feedback.

The proposed method is not only to identify a structural equation but also to assign a direct relationship to the physical components of the equation by using independent and dependent variables. The method parses the defined independent and dependent variables within equations and reconstructs the equations to be asymmetric causal equations. We use the term "asymmetric causal equation" instead of "structural equation", because a structural equation should express the "real" causality; our equations are structural-like, but express "reasonable" causality. Causality then can be easily assigned from the causal equations. If the dependency of variables can not be fully specified, then the method will generate all plausible behaviour.

Further, the system is able to be used for a model revision. It has the capabilities of allowing the users to edit the causal graph and make their new hypotheses. The system then backtracks through the proposed graph for consistency checking.

This paper presents an approach to specify causality within a feedback or non-feedback system. The method is generic and has been successfully tested on many domains. The construction of an asymmetric equation can constrain and derive the causal relationships among variables. Thus, causal dependencies are determined by other causal assignments within equations. It is our opinion that the notion of causality deduced from the proposed method is a context-dependent causality.

2. Causality

What is the concept of causality? There is no universal agreement on how to define causality or what it really is [Iwasaki 88a]. According to Bunge [Bunge 79], causation can be denoted as a connection between two events - *If X is a cause of Y, a change in X produces a change in Y and not that a change in X is followed by or associated with a change in Y.* Such a change can either be a discrete change or it can be a gradual change over an interval.

2.1 Independent Variables vs Dependent Variables

A *mathematical model* describes the dynamic behaviour of a system in terms of a set of non-redundant *equations*, and a set of *parameters*. A parameter is either a *variable* or a *constant*. The type of a variable is either *independent (or exogenous)* or *dependent (or endogenous)*. A dependent variable is a variable that is directly caused or influenced by any other variables. The value of dependent variable is to be determined by the interactions between the parameters in the model. An independent variable is a variable that is influenced from outside the system directly and produces a change to other variables. The value of independent variable is set by mechanisms that are outside the particular model under consideration. A *constant* is a parameter of the model whose value remains constant throughout the analysis. It can be treated as exogenous variable of a system.

Input and output information are available in many systems to state their entry situations and results. All input variables are independent variables whereas output variables are a subset of dependent variables.

However, when a system has feedback involved, causal analyses becomes very complicated. Some variables can both be dependent and independent. For example, figure 1 shows a causal graph of the brain control of glucoregulation model in which every variable is both a cause and an effect [Feldman 89]. It should be noted that this is a very simplified model of a glucose control system.

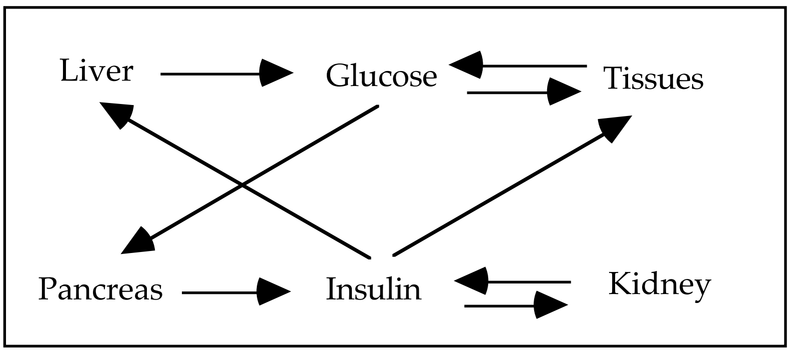

Figure 1. A causal graph of brain control of glucoregulation model.

It can be noted that pancreas, liver, tissues and kidney only have a single causal arrow leaving them, in some sense then they can be said to have a lesser or more localised causal effect than glucose and insulin which effect two or three other parameters respectively.

An important heuristic that we use is to reorganise equations by first of all propagating causality from the parameters which have least effect. We also start from the equation where there are the greatest number of such variables. This heuristic invariably produces the correct structural equation (in examples where we know what equation should be produced). We hypothesise that has something to do with parsimony in scientific explanation. The normal idea in building a model is to discover (create) the smallest number of entities and causal connections to explain the behaviour of the system. More important variables will be used as frequently as possible in the model, while other variables are discovered (created) to fill in the gaps. Our method deals with the causal influences from these "lesser" variables first. Our current research aims at clarifying what classes of problems this is suitable for and why. However we note it works on all of the problems noted below, including a very large Global Energy Model [Edmonds 83] with 41 equations and 73 parameters.

2.2 Causal Language vs Mathematical Language

It is important to distinguish the language interpretation in a mathematical world and a causal world when deducing causality from equations. One can transform equations so that variables move form one side of the equations to the other. However we use equations to represent structure in the world. Iwasaki's [Iwasaki 86, 88, 88a] definition of a structural equation is simply one that represents structure in the world. She gives no definition of how a structural equation should be expressed, and De Kleer and Brown have pointed out that equations cannot really express direct relationships to the physical components [de Kleer 86]. Hayduk [Hayduk 87] notes that if you are trying to represent the world, you only have a single variable on the LHS and the equation represents the causal effects on this model. Simply, a structural equation is the type of equation used to describe a physical system. In terms of causality, a caused or dependent variable will appear only once on the LHS of the equation, that is, each equation expresses all the causal influences on a particular parameter.

We suggest further that the normal convention is not only a single dependent equation on the left, but that each variable will appear no more than once on the left. In other words when you are building a causal model of the world you consider all the effects on a particular parameter together. Figure 2 shows equations for the above glucoregulation model.

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dglucose/dt = liver - tissues
dinsulin/dt = pancreas - kidney
tissues = c1 * insulin * glucose
kidney = c2 * insulin
liver = c3 * insulin
pancreas = c4 * glucose
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Figure 2. Equations of glucoregulation model

Six equations represent the dynamic behaviour of six dependent variables. There may be other forms of structural equations so we prefer to use the term asymmetric casual equations to describe such a set of equations. Although there may be other structural equations, the asymmetric causal form is a very common way of representing causality, so the aim of our heuristics is to reduce any set of equations to this form, in that it should then be reasonably intelligible to the user in terms of causality.

It is also important to notice that the interpretation of causality from mathematical equations is in abstracted description of a domain model. The precise quantitative information is not required in a causal model. The only concern is the relationships between variables where each variable has a non-zero coefficient within an equation [Iwasaki 88].

3. The Prototype System

We have built a prototype system, a **T**ool for **A**cquiring **K**nowledge from **E**quations (TAKE). This prototype has been developed in a Hypercard. It accepts a mathematical model in the form of equations, and from these equations deduces the structural equations and also produces easily maintained causal graphs. As well, a suite of tools are provided to allow the user to investigate the model(s) constructed. Usage to date indicates that this tool is a very practical. Figure 3 shows the structure of TAKE.

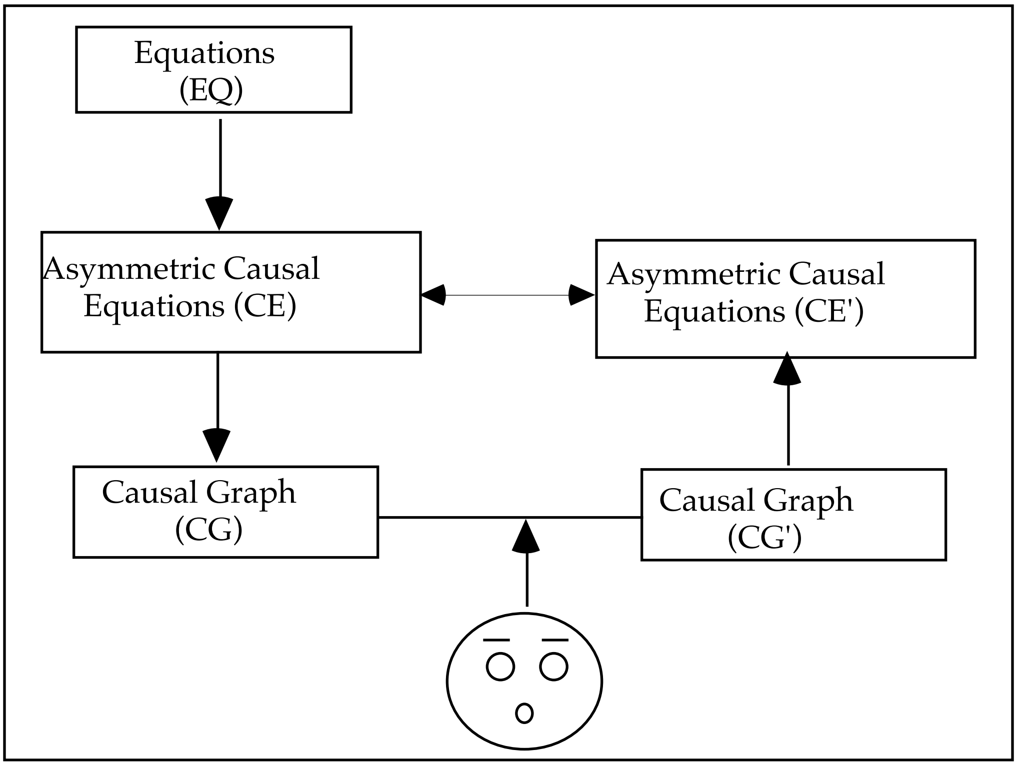

Figure 3. The structure of the TAKE tool.

Equations (EQ) are a finite set of simultaneous equations, which come from a mathematical model that describes the dynamic behaviour of the system. The tool identifies variable dependencies first, then restructures equations to be asymmetric causal equations (CE). Each CE model generates one causal graph, which represents a directed graph. Users can make new assumptions about the causality of the model, by manipulating the graph. Then TAKE will traverse back to build up a new set of asymmetric causal equation (CE').

During the generation of CE, the program parses variables within EQ, checking consistency between them. If the position of variables can not be determined, then the program will trace all possible combinations to generate *n* possible sets of CE. Each set of CE corresponds to one set of causal graphs. The assignment of causality in CE is from the RHS of an equation to the LHS of the equation, which indicates that the LHS variables depend on the RHS of the equation. Figure 4 shows the mapping relationships among processes.

Figure 4. The mapping relationships among process. The line with one chicken foot means "one-to-many" mapping. EQ to CE is a one-to-many mapping and CE to CG is a one-to-one mapping.

3.1 Identification of Dependency

The identification of variable dependency characteristics is defined by users or from heuristic assumptions. Inputs and outputs are known in some systems. Users and domain experts can predetermine inputs and outputs of a system. For instance, an electrical engineer recognises as obvious the input and outputs when designing a regulator circuit. Once inputs and outputs have been specified, then it is easy to investigate intermediate variables in the model.

Mathematical models offer a complementary relation among equations and describe the structure and function of systems [Kunz 89]. Often causal dependency is defined within equations. For instance in an equation where X = c1, X is a variable and c1 is a constant. If there is only one variable in an equation, then we define X as an exogenous variable. The value of X is determined only by variables that are outside the system [Iwasaki 86, 88a, Top 91].

However, sometimes it is impossible to identify input, especially in a feedback system, so that it is necessary to apply heuristics to determine the causality which will be identified.

3.2 Definition of Asymmetric Causal Equations

An asymmetric causal equation is an equation together with independent and dependent variables represented in an ordered form as stated in below. Implicit in the asymmetric is the notion that the variable on the LHS is dependent on the RHS, hence, the asymmetry of only one variable on the LHS of each equation. The definition of an asymmetric causal equation is:

(a) If an **output** variable appears on the LHS of the equation, it is the variable whose behaviour is of interest.
(b) An **input** or **independent** variable appears on the RHS of the equation.

(c) A **dependent** variable can be either side of an equation, with each dependent variable occurring once and only once on the LHS in the set of equations representing the model.
(d) The differential equation is a *canonical form* [Iwasaki 88]. The **derivative** is on the LHS of the equation, with only one derivative in each equation.

3.3 Dependency Heuristics

There may be a number of asymmetric causal forms for a given set of equations. If so the equations with the greatest number of least occurring variables are chosen. The equation is manipulated so that the least occurring variables are on the RHS. The remaining of the equations are then manipulated to give an asymmetric causal form. The first equation is now considered fixed and the process repeated for the rest of the equations etc.

3.4 An evaporator example

To further explain the method, consider the evaporator example in [Iwasaki 86]. Figure 5 shows an evaporator.

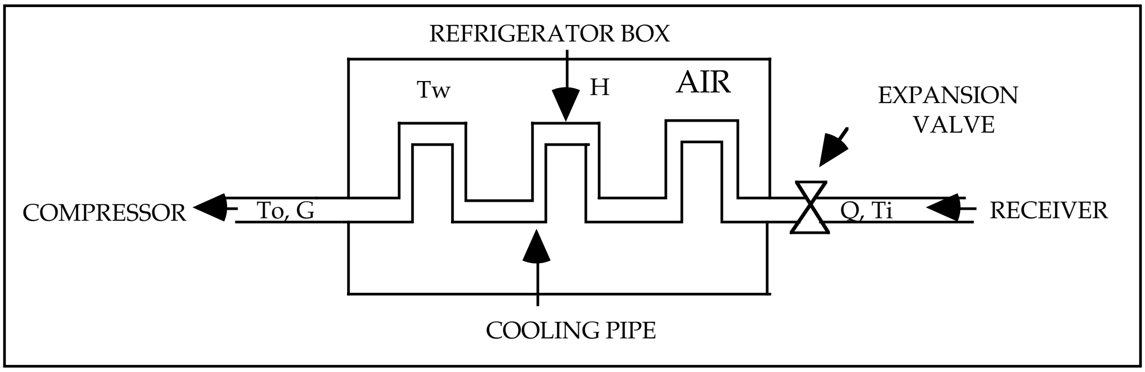

Figure 5. Evaporator (source from [Iwasaki 86]).

The variables are:

Q	refrigerant flow rate (mass/second),
Ti, To	temperatures of incoming and outgoing refrigerant,
G	ratio of vapour to total mass of outgoing refrigerant,
H	heat gained by the refrigerant,
P	pressure of refrigerant within chamber,
Tc	condensing temperature of refrigerant,
Tw	temperature of air in refrigerator chamber.

The equations, which describe the behaviour of an evaporator, are:

$$H = k(Tw-Tc) \quad (1)$$
$$H = GQl-(Ti-To)Qsp_1 \quad (2)$$
$$Tc = f1(P) \quad (3)$$
$$To = Tc \quad (4)$$

The constants are *spl, l,* and *k*.

According to the description of an evaporator, the user can state their interest of:

G and To to be the output variables in the system.

The dependency heuristic determines the lowest influence variables in the system as independent variables:

Q, Ti, P and Tw represent the independent variables in the system.

Q, Ti, P, and Tw, have less influence on the system, and are treated as exogenous variables in the causal ordering. The intermediate dependent variables are H and Tc.

We then propagate variables within the equations and generate the asymmetric causal equations of the evaporator model. The asymmetric causal equations of the evaporator model are:

$$H = k(Tw-Tc) \quad (1)$$
$$G = (H+(Ti-To)Qsp_1)/Ql \quad (2)$$
$$Tc = f1(P) \quad (3)$$
$$To = Tc. \quad (4)$$

The dependent variables are on the LHS of equations and appear once on each equation. Once we get this asymmetric causal structure of the model, we then determine a causal link for the evaporator. The procedure is specified by removing the constant variables first. Then causality in the evaporator can be simply stated as:

$$H \Leftarrow Tw, Tc \quad (1)$$
$$G \Leftarrow H, Ti, To, Q \quad (2)$$
$$Tc \Leftarrow P \quad (3)$$
$$To \Leftarrow Tc \quad (4)$$

The arrows mean that the LHS of variables depend on the RHS of variables. By using a graphical representation of above the causal links, figure 6 shows the causal graph of an evaporator.

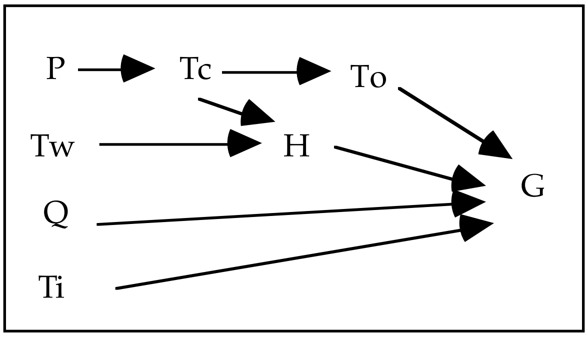

Figure 6. The causal graph of an evaporator.

In this example, our method and the causal ordering can produce identical causal links between variables. However, in terms of determining exogenous variables, we provide an automatic detection method to support modellers' decisions rather than rely on modellers.

3.5 Consistency Checking of New Hypotheses

The prototype system also allows users to make hypotheses. Users can make new assumptions about causality within the model. The system will begin with the causal graph generated by the user of the existing model, then traverse back to build up the asymmetric causal equations for justifying the hypothesis. Figure 7 shows a causal graph, which represents an hypothesis specified by a user.

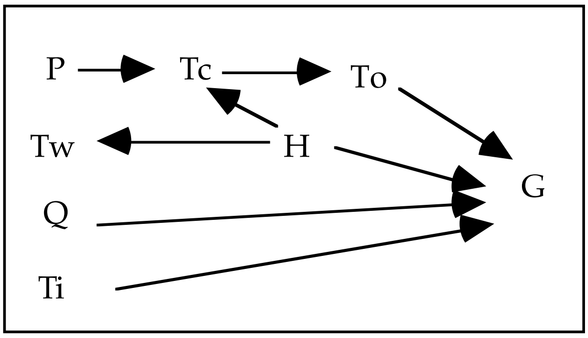

Figure 7. User hypothesis a causal graph of evaporator.

Based on the standard form of asymmetric causal equations, the LHS of the equation is for the concept of "effect" and RHS of the equation is for the concept of "cause". The above graph can represent asymmetric causal equations (SO') as:

Tc = [P], [H]
G = [Q], [H], [Ti], [To]
Tw = [H]
To = [Tc]

An important point: the generation of asymmetric causal equations from a causal graph uses qualitative representation. Tw = [H] means that the function of heat (H) causes the temperature of air in the refrigerator chamber (Tw) to change. By comparison with the defined dependencies of the original equations, we find that Tw becomes a dependent variable and H an independent variable. Also, there is some inconsistency between Tc and H, in terms of the dependency between Tc and H. Our intention is not to make a new scientific interpretation but to let users justify their hypotheses only.

4. Comparison with Related Research

4.1 Causal Ordering Theory

The theory of causal ordering defines *causal ordering* as an asymmetric relationship among variables in a set of simultaneous equations [Iwasaki 86]. The approach is based on the theory of causal ordering first presented by Simon in 1952.

4.1.1 Self-Containment and Exogenous Variables

The theory of causal ordering requires a *self-contained* structure to describe a system [Iwasaki 86, 88, 88a]. A self-contained structure is a system that has n equations and n unknowns. Unfortunately, in a real world model, it is seldom possible to meet these requirements. The detection of a self-contained system in causal ordering is up to users to carry out. To make a system self-contained, causal ordering also relies on the notion of exogenous variables. A variable that can be manipulated experimentally, for example, would be considered as an exogenous variable. To make a system into a self-contained structure and to assign exogenous variables relies upon an expert's experience and general knowledge of the model [Top 91].

In contrast, our approach does not require a system to have a self-contained structure. Equations used in the prototype system can use the same equations as in the simulation model without modifications. The method uses available input, output and any causal information from users, or detects variables automatically to be dependent or independent variables, and then assigns causality among variables.

4.1.2 Feedback

In the glucoregulation model in figure 2, equations are in a self-contained structure, six equations have exactly six unknowns. When we applied the theory of causal ordering to this model, the implementation of causal ordering eventually ceased. There is no minimal complete subset within equations, hence, there is no substitution between variables. No causal ordering among variables are indicated within the loop.

The prototype system identified the equations to be asymmetric causal equations, and then generated the causal graphs as in figure 1. Note we also manipulated the equations to mislead the system but the correct form was always discovered.

4.1.3 Consistency Checking

An acceptance of causal explanation is not just that it is constructed to be intuitively acceptable but rather

(i) the mechanisms on which it is built are consistent between equations and causal thinking,
(ii) these mechanisms are fully resolved.

It is desirable to confirm whether a theory has captured causality correctly. The causal ordering does *presume* that the equations used in the model are structural equations, and that the result of the causal ordering is in a correct form.

The prototype system provides a mechanism that defines the dependencies of variables first and then generates the equations in an asymmetric causal form. The use of the dependency characteristic of variables can restrict the number of possible causal pathways and detect inconsistencies between equations and causal reasoning.

4.1.4 Aggregation

When we study a mathematical model that involves many variables and relations, it becomes very complex to understand. Iwasaki [Iwasaki 88a] presented a method of aggregating the model to generate a more abstract model involving fewer variables and relations, and then produce causal ordering from the sub-model.

Our approach can capture all the knowledge without decomposition of the model. Once the asymmetric causal equations are generated by using the available knowledge of a system, the assignment of causality is simply obtained by pointing from the RHS of the equation to the LHS on each asymmetric causal equation. However, when experts give an explanation from the large number of causal links, we then aggregate the causal links to be a number of small groups and show the relatively important parts. We have applied the method to a Global Energy Model with 41 equations and 73 parameters, and generated 107 causal links. We then aggregated these causal links to 8 subgroups to show the relatively important subgroups of variables.

4.2 Bond Graphs

The Bond-Graph method uses a graphical approach to system modelling. The essential feature of the bond graph is the representation of energetic interactions between system components by a single line or energy bond. It has been used in the engineering world since the 1960's [Karnopp 75, Top 90, 91].

We note that Top has compared the Bond-Graph method to the theory of causal ordering [Top 91]. It may well be, as Top argued, that a bond-graph produces a clearer representation of causality than causal ordering from equations, especially in energy equations. However, to build the bond-graph requires a deep knowledge of the domain and a number of difficult modelling decisions. Our purpose is decision support when equations exist, but the decision maker does not understand the model, as in the Greenhouse Effect.

5. Conclusion

We have described a method for determining causal relationships of the behaviour of mathematical models. This method is based on applying heuristics to analysis the equations as well as using information from the user which possible.

We have compared the proposed method with the causal ordering theory of Iwasaki and Simon. Our method does not presume the equations used are structural. The method detects structural equations, and also produces equations to be asymmetric causal equations. The results of these two methods turn out to be similar, except in handling feedback and self-containment. The proposed method also produces a maximum causal resolution and automatically determines exogeneity, which makes knowledge acquisition easier.

We have successfully tested the method on more than 20 mathematical models including all of those in the relevant literature and in all cases the method discovers the "correct" causality. However, equations in mathematical models do not model any original causality. We need to further investigate what class of equations (if any) have implicit causality and what classes of equations do not. However, our heuristic does manage to recapture the original causality where relevant, or at least a reasonable causality where the equation was not based on an initial causal model.

The next stage of this research is to attempt to understand what classes of mathematical models this heuristic works or all in what way causality is implicit in these models. Also, it is essential to determine for what models the system gives causality that conflicts with the original causality the expert had in mind when devising the models. As well, we need to incorporate the concept of time in differential equations and to assign causal effects (+ or - signs). We also would like to manipulate the causal graph by simulating behaviours qualitatively. Kuipers' QSIM [Kuipers 86] approach produces too many behaviours to deal with. However, if we can limit the input to an acceptable causal graph, then maybe we can limit the behaviour generation.

Acknowledgments

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